Probability of Outcomes of A&A Battles

INTRODUCTION

This paper describes an algorithm for the explicit calculation of the probabilities of the outcomes of Axis and Allies Battles. Before reading this paper you must understand the Axis and Allies battle resolution system. I refer you to the second edition rule book by Avalon Hill.

The Axis and Allies game has become one of the most popular World War II themed board games. It also adapts well as a play-be-email game. Several play-by-email Axis and Allies clubs have become very well organized using the World Wide Web. Members of these clubs have developed utilities to improve their game. For instance, many players use battle simulators to help determine whether a given attack is advisable or inadvisable.

A battle simulator uses a Monte Carlo method to estimate the probabilities of the various outcomes of a hypothetical Axis and Allies battle. Typically, a simulator might run a given battle 1000 times. (The computer uses a pseudo random number algorithm to simulate dice.) At the end of every battle the computer records the winner and the number and type of surviving units. At the end of the 1000 battles, the computer divides the numbers in the registers by 1000 to estimate the probabilities of the various outcomes. For instance, if the attacker wins 612 of the 1000 battles, then the computer estimates the probability of the attacker winning as 0.612, or 61.2%. Estimates of the number of surviving units may be made in a similar fashion. Given a large enough number of simulated battles, the Weak Law of Large Numbers suggests that the estimate is sound. This Monte Carlo method is easy to implement on a digital computer, and several different battle simulators are available for free on the World Wide Web.

By contrast, a battle calculator tabulates the probabilities explicitly using the attack and defense probabilities of each unit in the battle, and algorithms and formula resulting from the Axis and Allies battle process. A battle calculator is more complicated than a battle simulator, because even the simplest battle possesses an infinitely large probability space. For example, consider the well known battle between one attacking armor and one defending infantry. In the first round either or both of the units might be destroyed. But also, both units might survive – the probability is 1 in 9, or about 11%. Then in the second round, both units might survive a second time – again the probability is 1 in 9. But the chance of both units surviving both rounds is the intersection of both probabilities – 1 in 81, or about 1.1%. Similary, there exists a chance that both units might survive 10 rounds. The chance is very small – 1 in 3,486,784,401. But as this paper will show, these very small probabilities form a geometric sequence, whose sum is a significant and easily calculable number.

DEFINITIONS

Unit

A unit is one of the following: infantry, artillery, marines, armor, fighter, bomber, submarine, transport, destroyer, battleship, or aircraft carrier.

Unit Type

There are twelve types of units. Units of the same type are identical. A unit type contains the following data elements:

- type_name. Character string; one of the following: “inf”, “inf2”, “arty”, “mar”, “mar2”, “arm”, “ftr”, “bmb”, “tn”, “sub”, “dst”, “BB”, “BBd”, “AC”.
- attack_factor. This property is an integer which represents six times the probability that the unit will destroy an enemy when this unit type is attacking.
- defence_factor. This property is an integer which represents six times the probability that the unit will destroy an enemy when this unit type is defending.

type_id	0	1	2	3	4	5	6	7	8	9	10	11	12	13
name	inf	inf2	mar	mar2	arty	arm	ftr	bmb	tn	sub	dst	BB	BBd	AC
att_factor	1	2	2	3	2	3	3	4	0	2	3	4	4	1
def_factor	2	2	2	2	2	2	4	1	1	2	3	4	4	3
is_land	1	1	1	1	1	1	0	0	0	0	0	0	0	0
is_air	0	0	0	0	0	0	1	1	0	0	0	0	0	0
is_naval	0	0	0	0	0	0	0	0	1	1	1	1	1	1

Table 1: Unit Types

- is_land. This property is a boolean. If the unit type is one of {"inf", "arty" or "arm"} then this property is true, otherwise it is false.
- is_air. This property is a boolean. If the unit type is one of { "ftr" or "bmb" } then this property is true, otherwise it is false.
- is_naval. This property is a boolean. If the unit type is one of { "tn", "sub", "dst", "BB", "BBd", or "AC"} then this property is true, otherwise it is false.

Army

An army is a group of units.

Battle

A battle occurs when one army attacks another army.

Order of Loss (OOL)

During a battle units may be lost (i.e. removed from play). Order of loss is the order in which units are removed from play.

Unit List

The units belonging to an army may be arranged sequentially in reverse order of loss. For instance, if there are twenty units in the army, the first unit in the list will be the last unit lost. The last unit in the list will be the first unit lost. When an army incurs losses during a battle, units are removed from the end of the attacker's unit list.

Standard Battle

A standard battle is a battle which conforms to the following conditions:

- no submerging
- no submarine first shot
- no anti-aircraft guns
- all attacking units may hit all defending units, and vice versa
- no shore bombardment
- no retreats
- OOL is set at the beginning of the battle and cannot change during the battle

Round

In an A&A game battles are resolved as a series of one or more rounds. A round is an instance of combat. During a round, all attackers fire and all defenders return fire. The results of the firing determines the number of attacking and defending units lost. The round ends after losses are removed from the attacking and defending armies. We assume the conditions of a standard battle.

FINITE STATE MACHINE MODEL

This analysis will model an axis and allies battle as a state machine. The machine starts at a state where both armies are full strength. The state machine includes all states of lesser strength. Then the machine moves from one state to another based on rolls of the dice. The machine stops when it reaches a terminal state in which one or both armies have been destroyed. Each possible transition between states has a definite, calculable probability. Therefore in theory it should be possible to calculate the probability of arriving in a given terminal state. There are two complications: first, there exist loops in the state machine corresponding to instances when the attacker and defender both roll zero hits. Second, there exist multiple paths through the state machine to a some terminal states. This paper develops answers to both these problems.

BATTLE NOTATION

Initial Battle Information

A battle consists of an attacking army and a defending army. The attacking unit list and defending unit list must be specified at the beginning of a battle, for instance:

```
Battle #1
att_list = { BMB, FTR, ARM, ARTY, INF2, INF }
def_list = { FTR, INF, INF, INF }
```

Battle State Notation

During a battle the number of units in each army decreases. Losses are always removed from the end of the unit list, but the order of units in the unit list always stays the same. The list was defined at the beginning of the battle. Therefore it is not necessary to list the units remaining, we can fully describe the surviving units simply by stating the number of units remaining in the attacking and defending armies. This is done using the BS prefix followed by the number of attacking and defending units, in parenthesis. For instance,

```
Battle #1
att_list = { BMB, FTR, ARM, ARTY, INF2, INF }
def_list = { FTR, INF, INF, INF }
BS(3,2)
```

defines a battle state wherein the attacking army contains a bomber, a fighter and an armor unit and the defending army consists of a fighter and an infantry.

Battle State Transition

Each round of battle starts with one state and ends with another states. The move from one state to another is a battle state transition. In fact, if neither army scores any hits, the starting and ending states may be the same. Even so, a transition from one state to the same state is still considered to be a battle state transition. A battle state transition is noted by writing the initial battle state, followed by a "->" sign, concluding with the end battle state.

Range of Battle State Transitions

Given a certain initial battle state, there are a limited number of final battle states. The initial state, followed by the list of all possible final battle states, represents the range of battle state transitions. The range of battle state transitions for a given battle state BS(x,y) may be represented by the expression R(BS(x,y)). This expression may be used in a summation, as will be demonstrated in the next definition.

Battle State Transition Probability

The chance that a specific battle state transition might occur is the battle state transition probability, expressed as P(BS (a,b)->BS(c,d)). By definition, the summation of battle state transition probabilities over the entire range of battle state transitions for a given battle state is 1.

CALCULATION OF BATTLE STATE TRANSITION PROBABILITY

Any battle state transition probability may be calculated. This section describes a simple minded algorithm which may be used to perform the calculation. I call it simple minded because it scales as the factorial of the number of units involved in the battle. Therefore the calculation becomes too complex for large battles. However this calculation provides a starting point that others may improve upon.

Attacker Hits and Defender Hits

Consider the battle state transition given by

```
Battle #1
att_list = { BMB, FTR, ARM, ARTY, INF2, INF }
def_list = { FTR, INF, INF, INF }
BS(6,4) -> BS(4,3)
```

Getting back to the basics of the game, this transition expressses a situation where the attacker has hit one defender and the defender has hit two attackers. For this section I will use a shorthand notation. Six attackers make 3 hits will be anotated like 6a3. Four defenders make two hits will be anotated as 4d2.

Independence of Attack and Defence

The number of attacker hits is independent of the number of defender hits. Therefore the probability may be expressed as:

P(BS(6,4)->BS(4,3)) = P(6a1) x P(4d2)

So the problem of calculating battle state transition probability is equivalent to the problem of calculating the probability that **x** units score **t** hits.

Probability That x Units Score t Hits

Suppose that an army of six attacking units scores 3 hits (6a3). It could be that the first three units score hits and the last three units miss, or the first three units might have missed and the last three units hit. Or there could be many other combinations whereby six units score three hits. In fact, this is a common combinatorics problem, called "six choose three" or written in mathematical notation as

$$\binom{6}{3}$$

The value of **x** choose **t** can be calculated using factorials, given the following formula:

$$\binom{x}{t} = \frac{x!}{t!(x-t)!} \quad ; \quad \binom{6}{3} = \frac{6\times5\times4\times3\times2\times1}{(3\times2\times1)(3\times2\times1)} = 5\times4 = 20$$

It is very informative to list the possible combinations. Suppose that the six units are ordered from one to six. Suppose "1" represents a hit and "0" represents a miss. Then the code 111000 represents the first three units hitting and the last three missing. Using this representation, the twenty possible combinations are:

```
1:  111000
2:  110100
3:  110010
4:  110001
5:  101100
6:  101010
7:  101001
8:  100110
9:  100101
10: 100011
11: 011100
12: 011010
13: 011001
14: 010110
15: 010101
16: 010011
17: 001110
18: 001101
19: 001011
20: 000111
```

Now we can calculate any probability **x** hits **t**. Here's how:

1. We know the attack and defence values of the units, so the probability of a given combination occuring is easy to calculate: for instance, if the first three units are armor and the last three units are infantry, and the units are attacking, then the probability of combination number 15 (010101) is shown by the table below. The top line shows the code representing the hits. The second line shows the probability of missing and hitting, for each unit. If the unit misses ("0"), then we follow the arrow to the miss probability. If the unit hits ("1") then we follow the arrow to the hit probability. Then we multiply the probabilities together to get the probability of the individual combination. In this case, the probability is .0028, or .28%.

0	1	0	1	0	1

3/6	3/6	3/6	3/6	3/6	3/6	5/6	1/6	5/6	1/6	5/6	1/6

$$\frac{3}{6}\times\frac{3}{6}\times\frac{3}{6}\times\frac{1}{6}\times\frac{5}{6}\times\frac{1}{6}=\frac{135}{46656}=.00289$$

2. We know that the probability that x hits t is just the sum of the probabilities of each combination. The following box shows the probabilities for each of the twenty combinations of 6a3, given three armor and three infantry. The total probability is 10666/46656, which is about 23%.

```
1:  111000  P = 3375/46656
2:  110100  P =  675/46656
3:  110010  P =  675/46656
4:  110001  P =  675/46656
5:  101100  P =  675/46656
6:  101010  P =  675/46656
7:  101001  P =  675/46656
8:  100110  P =  135/46656
9:  100101  P =  135/46656
10: 100011  P =  135/46656
11: 011100  P =  675/46656
12: 011010  P =  675/46656
13: 011001  P =  675/46656
14: 010110  P =  135/46656
15: 010101  P =  135/46656
16: 010011  P =  135/46656
17: 001110  P =  135/46656
18: 001101  P =  135/46656
19: 001011  P =  135/46656
20: 000111  P =    1/46656
```

3. Finally, we need a way to generate the list of combinations for any x hits t. The method is simple: any computer programmer looking at the listing of twenty combinations for 6 hits 3 will notice the pattern. Simply start with "t" hits, followed by (x-t) misses. Then shift the last hit left, generating a new combination with each shift operation until the last hit is in the end of the bitfield. Then move the hit back to its original position, and shift two hits to the right. Then repeat the previous shifting operation. A simple algorithm to generate an appropriate set of bitfields for any x hits t is given in Appendix A.

$$P(BS(a,d)\rightarrow BS(x,y))=P_{a,d}(x,y)$$

$$P_{a,d}(x,y)=(\sum P_{combination}(a\ hits(d-y)))(\sum P_{combination}(d\ hits(a-x)))$$

Formula 1: Formula for Battle State Transition Probability

Formula for Battle State Transition Probability

Figure 1 shows the formula for transition state probability. The summation is the sum of the probabilities for each combination. The first summation is for the attackers' kills, the second part is for the defenders'.

In theory this calculation may be performed for any battle state transition. In reality, the number of combinations and therefore the time required to perform the calculation, increases dramatically as the number of units involved increases, as shown by the table below. The time estimate assumes about 1000 instructions to calculate each probability and a processor speed of about 3000 mips.

x	*t*	*# Combinations*	*Time*
4	2	6	1.3 us
8	4	70	23 us
12	6	924	0.3 ms
16	8	12,870	4.3 ms
20	10	184,756	61 ms
24	12	2,704,156	0.9 s
28	14	40,116,600	13 s
32	16	601,080,390	3.3 minutes
36	18	9,075,135,300	50.4 minutes
40	20	137,846,528,820	12.7 hours

BATTLE STATE TRANSITION DIAGRAMS

Simplest Possible Battle

Consider the simplest battle: 1 attacking unit vs 1 defending unit. In the first round of battle there are four possible battle state transitions: BS(1,1)->BS(1,1), BS(1,1)->BS(1,0), BS(1,1)->BS(0,1) and BS(1,1)->BS(0,0). Three of these transitions result in terminal battle states - that is, one or both armies have been destroyed and the battle is over. But the first battle state transition results in the same battle state, and therefore there will be a second round. If the second round ends in the same battle state, then there will be a third round, and so on. Figure 1 illustrates the battle state transitions.

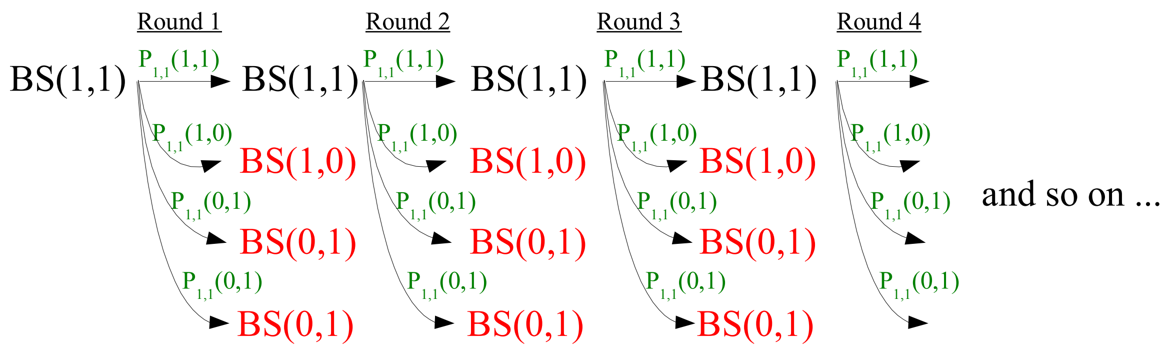

Figure 1: Battle State Transitions for simplest battle

The probabilities of making each transition are shown in Figure 1 in green. For instance, the probability of arriving in BS (1,0) in round 1 is $P_{1,1}(1,0)$. Here is a more interesting question: what is the probability that the battle will end in BS(1,0) at the end of the second round? This probability is simply the product of the probability of the state transition in the first round ($P_{1,1}(1,1)$) times the probability of the state transition in the second round ($P_{1,1}(1,0)$). What is the probability of arriving in BS(1,0) at the end of the third round? It is simply the product of arriving in BS(1,1) in the first round, times the probability of arriving at BS(1,1) at the end of the second round, times the probability of arriving at BS(1,0) at the end of the third round. The arrow in figure 2 illustrates an example path through a map of all possible battle states.

By

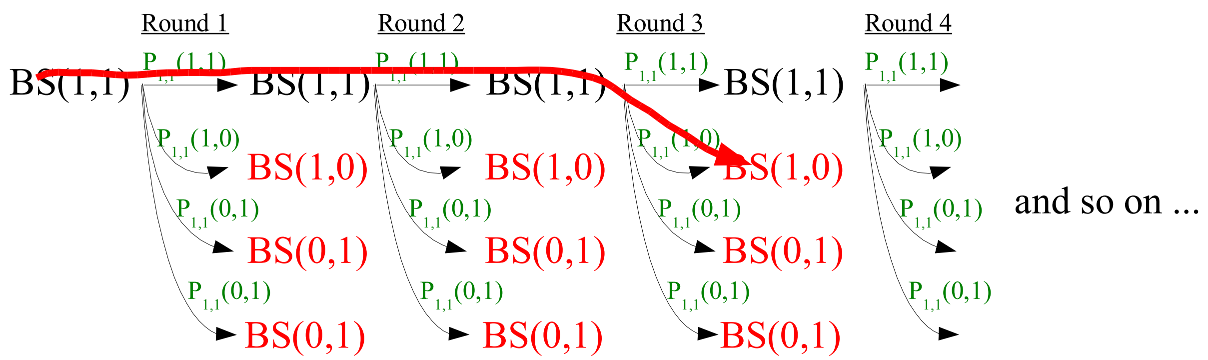

Figure 2: Path of a specific battle through a battle state diagram

$$P_{1,1}(1,1)P_{1,1}(1,1)P_{1,1}(1,0)=P^2{}_{1,1}(1,1)P_{1,1}(1,0)$$

following the arrow in figure 2, we see that the probability of this specific incident occuring is equal to $[P_{1,1}(1,1)]^2 P_{1,1}(1,0)$. The **Terminal Probability** is the total probability that the battle will end in a certain state. The capital letter T will represent the Terminal Probability. Therefore, the terminal probability that BS(1,1) will result in BS(1,0) is $T_{1,1}(1,0)$ which is the sum of the probability that the battle will end in BS(1,0) in round 1, plus the probability that the battle will end in BS(1,0) in round two, plus the probability that the battle will end in BS(1,0) in round three, and so on for an infinite number of rounds, as given in the following formula:

$$T_{1,1}(1,0) = P_{1,1}(1,0)+P_{1,1}(1,1)P_{1,1}(1,0)+P^2{}_{1,1}(1,1)P_{1,1}(1,0)+P^3{}_{1,1}(1,1)P_{1,1}(1,0)+\ldots$$

$$T_{1,1}(1,0) = P_{1,1}(1,0)[1+P_{1,1}(1,1)+P^2{}_{1,1}(1,1)+P^3{}_{1,1}(1,1)+\ldots]$$

$$\textit{we know that if } x<1 \textit{ then}: \quad 1+x+x^2+x^3+\ldots = \frac{1}{1-x}$$

$$\textit{therefore} \quad T_{1,1}(1,0) = P_{1,1}(1,0)\frac{1}{1-P_{1,1}(1,1)} = \frac{P_{1,1}(1,0)}{1-P_{1,1}(1,1)}$$

We see that the infinite series of probabilities can be simplified using the formula for the geometric series. Likewise, we can calculate the formulae for the other two terminal battle states:

$$T_{1,1}(0,1)=\frac{P_{1,1}(0,1)}{1-P_{1,1}(1,1)}$$

$$T_{1,1}(0,0)=\frac{P_{1,1}(0,0)}{1-P_{1,1}(1,1)}$$

Second Most Simple Battle

The second most simple battle is two attackers vs one defender (we will adopt the convention of n+1 attackers vs n defenders being "more simple" than n attackers vs n+1 defenders). Similar to the preceding section, we can develop a state transition diagram as follows:

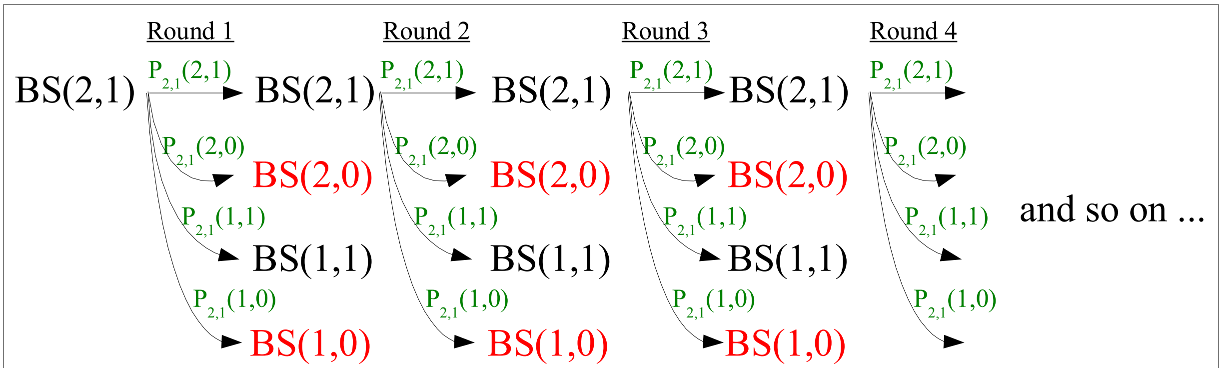

Figure 3: Battle State Transition Diagram for second most simple battle

BS(2,0) and BS(1,0) are obviously terminal battle states, but BS(1,1) is not. BS(1,1) is an intermediate battle state. The previous section analyzed BS(1,1) and we know that its terminal states are BS(1,0), BS(0,1), and BS(0,0) with probabilities given in the preceding section. When we calculate the terminal probability there is a term in the expression which represents the probability of the direct result, ie BS(2,1)->BS(1,0). And then there is a term for each intermediate result, ie. BS(2,1)->BS(1,1)->BS(1,0). Continuing this example, the term representing the probability that BS(2,1) moves directly to BS(1,0) is:

$$T_{2,1}(1,0)=\frac{P_{2,1}(1,0)}{1-P_{2,1}(2,1)} \quad + \quad \ldots$$

The intermediate state BS(1,1) contributes a second term to the terminal probability. This second term is the probability of moving to the BS(1,1) state times the probability of moving from BS(1,1) to BS(1,0), which is $T_{1,1}(1,0)$:

$$T_{2,1}(1,0)=\frac{P_{2,1}(1,0)}{1-P_{2,1}(2,1)} \quad + \quad \frac{P_{2,1}(1,1)}{1-P_{2,1}(2,1)}\times T_{1,1}(1,1)$$

In general, we can state that each intermediate state contributes an additional term for each terminal state which is accessible from that intermediate state. So for instance, the formulas for the terminal probabilities $T_{2,1}(1,0)$, $T_{2,1}(0,1)$, and $T_{2,1}(0,0)$ each contain a term from the BS(1,1) intermediate state:

$$T_{2,1}(2,0)=\frac{P_{2,1}(2,0)}{1-P_{2,1}(2,1)}$$
$$T_{2,1}(1,0)=\frac{P_{2,1}(1,0)}{1-P_{2,1}(2,1)}+\frac{P_{2,1}(1,1)}{1-P_{2,1}(2,1)}\times T_{1,1}(1,0)$$
$$T_{2,1}(0,1)=\frac{P_{2,1}(0,1)}{1-P_{2,1}(2,1)}+\frac{P_{2,1}(1,1)}{1-P_{2,1}(2,1)}\times T_{1,1}(0,1)$$
$$T_{2,1}(0,0)=\frac{P_{2,1}(0,0)}{1-P_{2,1}(2,1)}+\frac{P_{2,1}(1,1)}{1-P_{2,1}(2,1)}\times T_{1,1}(0,0)$$

And since $P_{2,1}(0,1)$ and $P_{2,1}(0,0)$ are both equal to zero, the equations simplify to:

$$T_{2,1}(2,0)=\frac{P_{2,1}(2,0)}{1-P_{2,1}(2,1)}$$
$$T_{2,1}(1,0)=\frac{P_{2,1}(1,0)}{1-P_{2,1}(2,1)}+\frac{P_{2,1}(1,1)}{1-P_{2,1}(2,1)}\times T_{1,1}(1,0)$$
$$T_{2,1}(0,1)=\frac{P_{2,1}(1,1)}{1-P_{2,1}(2,1)}\times T_{1,1}(0,1)$$
$$T_{2,1}(0,0)=\frac{P_{2,1}(1,1)}{1-P_{2,1}(2,1)}\times T_{1,1}(0,0)$$

It is possible to expand $T_{1,1}(1,0)$, $T_{1,1}(0,1)$ and $T_{1,1}(0,0)$ in terms of transition probabilities, but it is not very useful because as I will show in a later section, in makes more sense to calculate and store the terminal probabilities of the simple battles first, then plug these values into the formulae for the more complicated battles.

Probabilities for BS(1,2)

Using the same procedure as in the previous section, we can draw a battle state transition diagram as shown in figure 4. Figure 4 looks very similar to figure 3. And the calculation of probabilities is very much the same:

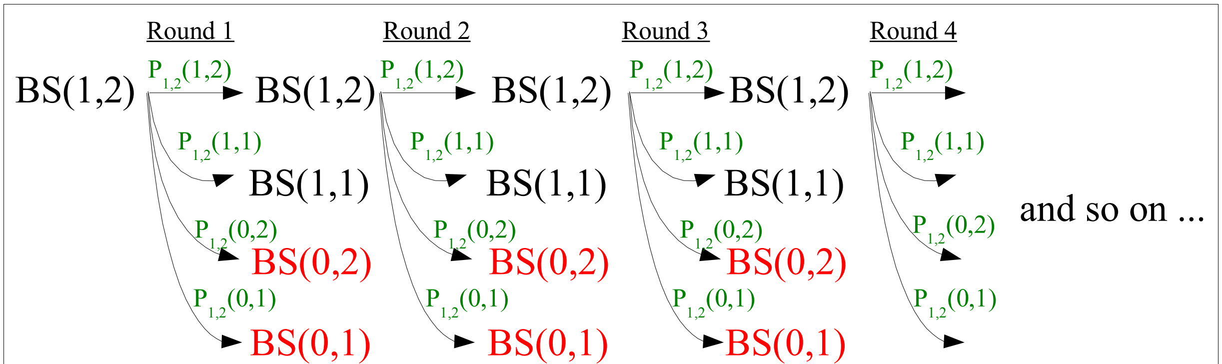

Figure 4: Battle State Transition Diagram for BS(1,2)

$$T_{1,2}(1,0)=\frac{P_{1,2}(1,1)}{1-P_{1,2}(1,2)}\times T_{1,1}(1,0)$$

$$T_{1,2}(0,0)=\frac{P_{1,2}(1,1)}{1-P_{1,2}(1,2)}\times T_{1,1}(0,0)$$

$$T_{1,2}(0,1)=\frac{P_{1,2}(0,1)}{1-P_{1,2}(2,1)}+\frac{P_{1,2}(1,1)}{1-P_{1,2}(1,2)}\times T_{1,1}(0,1)$$

$$T_{1,2}(0,2)=\frac{P_{1,2}(0,2)}{1-P_{1,2}(1,2)}$$

Probabilities for BS(2,2)

In the preceding sections we examined simple battles where one or both army consisted of only one unit. The case of BS (2,2) gives a hint of what a more complex (m,n) example would look like. Figure 5 shows the battle state transition diagram for BS(2,2).

Figure 5: Battle State Transition Diagram for BS(2,2)

As we can see, the number of battle state transitions has increased dramatically. In fact, we can make a general statement regarding the number of transitions for BS(m,n):

$$\textit{if } m > n \textit{ then } \text{number of transitions} = (n+1)^2$$
$$\textit{else } \text{number of transitions} = (m+1)^2$$

But even though the number of transitions has increased, the basic procedure established in the preceding sections stays the same. That is to say, the probability of a terminal state is equal to the direct probability of the terminal state plus the probability of the terminal state given that one of the intermediate states has occured. And we have already calculated all the probabilities for the intermediate states (BS(2,1), BS(1,2), and BS(1,1)). So we can determine the probabilities for each of the terminal states:

$$T_{2,2}(2,0) = \frac{P_{2,2}(2,0)}{1-P_{2,2}(2,2)} + \frac{P_{2,2}(2,1)}{1-P_{2,2}(2,2)} \times T_{2,1}(2,0)$$
$$T_{2,2}(1,0) = \frac{P_{2,2}(1,0)}{1-P_{2,2}(2,2)} + \frac{P_{2,2}(2,1)}{1-P_{2,2}(2,2)} \times T_{2,1}(1,0) + \frac{P_{2,2}(1,2)}{1-P_{2,2}(2,2)} \times T_{1,2}(1,0) + \frac{P_{2,2}(1,1)}{1-P_{2,2}(2,2)} \times T_{1,1}(1,0)$$
$$T_{2,2}(0,0) = \frac{P_{2,2}(0,0)}{1-P_{2,2}(2,2)} + \frac{P_{2,2}(2,1)}{1-P_{2,2}(2,2)} \times T_{2,1}(0,0) + \frac{P_{2,2}(1,2)}{1-P_{2,2}(2,2)} \times T_{1,2}(0,0) + \frac{P_{2,2}(1,1)}{1-P_{2,2}(2,2)} \times T_{1,1}(0,0)$$
$$T_{2,2}(0,1) = \frac{P_{2,2}(0,1)}{1-P_{2,2}(2,2)} + \frac{P_{2,2}(2,1)}{1-P_{2,2}(2,2)} \times T_{2,1}(0,1) + \frac{P_{2,2}(1,2)}{1-P_{2,2}(2,2)} \times T_{1,2}(0,1) + \frac{P_{2,2}(1,1)}{1-P_{2,2}(2,2)} \times T_{1,1}(0,1)$$
$$T_{2,2}(0,2) = \frac{P_{2,2}(0,2)}{1-P_{2,2}(2,2)} + \frac{P_{2,2}(1,2)}{1-P_{2,2}(2,2)} \times T_{1,2}(0,2)$$

These formula demonstrate that explicit calculation of complex battles can be accomplished by first calculating the outcomes of the simpler intermediate battles. In fact, an algorithm would start by calculating the terminal probabilities of the simplest battle, the one on one scenario, then progress incrementally by adding one unit to each of the attacking and defending armies, until it reaches the desired initial battle state. This sort of tedious but predictable caclulation is well suited to digital computers, and this paper will present a recursive algorithm. But first I will present a simple example, calculated by hand.

EXAMPLE BATTLE: 2 ARMOR VS 2 INFANTRY

To illustrate the method, consider 2 armor attacking 2 infantry. First it is necessary to calculate the terminal probabilities of the intermediate battles, starting with the most simple and increasing until we get to the actual battle of interest.

Intermediate Battle: 1 armor vs 1 infantry

Transition Probabilities		***Terminal Probabilities***	
$P_{1,1}(1,1)$	1/3	$T_{1,1}(1,0)$	1/2
$P_{1,1}(1,0)$	1/3	$T_{1,1}(0,0)$	1/4
$P_{1,1}(0,1)$	1/6	$T_{1,1}(0,1)$	1/4
$P_{1,1}(0,0)$	1/6		

Intermediate Battle: 2 armor vs 1 infantry

Transition Probabilities		***Terminal Probabilities***	
$P_{2,1}(2,1)$	1/6	$T_{2,1}(2,0)$	3/5
$P_{2,1}(2,0)$	1/2	$T_{2,1}(1,0)$	7/20
$P_{2,1}(1,1)$	1/12	$T_{2,1}(0,0)$	1/40
$P_{2,1}(1,0)$	1/4	$T_{2,1}(0,1)$	1/40

Intermediate Battle: 1 armor vs 2 infantry

Transition Probabilities		***Terminal Probabilities***	
$P_{1,2}(1,2)$	2/9	$T_{1,2}(1,0)$	1/7
$P_{1,2}(1,1)$	2/9	$T_{1,2}(0,0)$	1/14
$P_{1,2}(0,2)$	5/18	$T_{1,2}(0,1)$	3/7
$P_{1,2}(0,1)$	5/18	$T_{1,2}(0,2)$	5/14

Final Battle: 2 armor vs 2 infantry

Transition Probabilities		***Terminal Probabilities***	
$P_{2,2}(2,2)$	1/9	$T_{2,2}(2,0)$	11/40
$P_{2,2}(2,1)$	2/9	$T_{2,2}(1,0)$	199/560
$P_{2,2}(2,0)$	1/9	$T_{2,2}(0,0)$	61/560
$P_{2,2}(1,2)$	1/9	$T_{2,2}(0,1)$	207/1120
$P_{2,2}(1,1)$	2/9	$T_{2,2}(0,2)$	17/224
$P_{2,2}(1,0)$	1/9		
$P_{2,2}(0,2)$	1/36		
$P_{2,2}(0,1)$	1/18		
$P_{2,2}(0,0)$	1/36		

The final probabilities are:

- attacker wins with 2 armor: 11/40 or exactly 27.5%
- attacker wins with 1 armor: 199/560 or about 35.5%
- defender wins, no survivors: 61/560 or about 10.9%
- defender wins with 1 infantry: 207/1120 or about 18.5%
- defender wins with 2 infantry: 17/224 or about 7.6 %